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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE MAGNITUDE OF AN ERROR WHICH SOMETIMES AFFECTS THE RESULTS OF MAGNETIC TESTS UPON IRON AND STEEL RINGS.

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The theory of the magnetic properties of a homogeneous ring of iron or steel uniformly wound about by turns of insulated wire through which a steady current of electricity can be made to pass, was first investigated by Kirchhoff, who showed that the intensity of the magnetic field in the metal which thus forms the core of a ring solenoid, must be inversely proportional to the distance from the axis of revolution of the ring. He computed the mean value of the field in a ring of rectangular cross section,[1] and pointed out the advantages which rings offer for measurements of the magnetic permeabilities of the metals of which they are made. The next year, Stoletow, working under the advice of Kirchhoff, took up the subject practically in the Physical Laboratory of the University of Heidelberg and in 1872 published the results of a long series of experiments upon a ring forged from a wrought iron rod. In 1873 appeared an account of the important work of Rowland, begun three years before, on rings (toroids) of circular cross section, made of various kinds of iron and steel, and since that time countless measurements of permeability have been made by many observers[2] upon iron and steel rings; and when these rings have been turned out of masses of solid metal, and not forged up and welded from bars, the results have usually been satisfactory. The value of the mean intensity of the magnetic force within the mass of a ring of circular cross section was given without proof by Bauer in 1880; a proof was printed by Lehmann in 1893, and an interesting diagram based on the formulas of Kirchhoff and Bauer, and showing the ratio of the mean magnetizing force to the value of the force at the mean radius for rings and toroids of different relative dimensions, was given by Morton[3] in the Bulletin of the Bureau of Standards for February, 1909.

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[1] Gustav Kirchhoff, Pogg. Ann. Ergzbd. **5**, 1 (1870). Gesammelte Abhandlungen, 223; A. Stoletow, Pogg. Ann. **146**, 442 (1872); H. Rowland, Phil. Mag. **46**, 140 (1873); **48**, 321 (1874); C. Bauer, Wied. Ann. **11**, 349 (1880).

[2] J. A. Ewing, Magnetic Induction in Iron and other metals; G. vom Hofe, Wied. Ann. **37**, 482 (1889); H. Lehmann, Wied. Ann. **48**, 406 (1893); A. von Ettingshausen, Wied. Ann. **8**, 554 (1879); H. du Bois, Magnetische Kreise, 110 *et seq.* Berlin, 1894; L. Boltzmann, Anz. d. Wiener Akademie, 203 (1878); J. Sauter, Wied. Ann. **62**, 85 (1897); L. Mues, Inaug. Diss. Greifswald (1893); I. Schuetz, Journal f. Mathematik, **113**, 161 (1894); Carl Neumann, Ueber Ring Potentiale, Halle (1864).

In determining the permeability of an iron ring it is usual to demagnetize the metal as thoroughly as possible at the outset, and then, either by the "Method of Ascending Reversals" or the "Step-by-step Method" to determine for each of a number of values of the magnetomotive force, the whole flux of magnetic induction through the ring. The ratio (B') of this flux to the area of the cross section of the ring is then plotted against the mean value of the magnetic force in the metal to get an HB diagram for the given magnetic journey of the iron. It is clear, however, as the earliest workers in this field saw, that the process here described is only approximately exact, for the induction often has very different values at the points of the ring nearest the axis of revolution and at those farthest away from it. Indeed, in a ring of soft iron of the dimensions of the specimens employed in a well-known form of commercial testing apparatus used in Europe, the value of B at points on the inner edge of the ring when the average value of the force in the metal is unity, may be as high as 2,000, while the value at points at the outer surface is only 700. In this case there is a considerable difference between the average value (B') of the flux in the metal and the real value (B'') of B at points of the ring where H has the average value. For relatively slender rings and fairly high excitations the discrepancy is not so great, and various attempts have been made to estimate its amount beforehand for materials of different kinds. It sometimes happens, however, that one has at command only a small piece of the iron to be tested, and it becomes necessary to make the measurements upon a relatively stout ring not much larger than a finger ring, as Dr. A. Campbell of the National

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[3] J. A. Moellinger, Elect. Zs., **22**, 379 (1901); B. Soschinski, Elect. Zs., **24**, 292 (1903); J. W. Esterline, Proc. Am. Soc. for Testing Materials, **3**, 288 (1903); R. Richter, Elect. Zs., **24**, 710 (1903); R. Edler, Mitt. Techn. Gewerbe-Museums, Vienna, **16**, 67 (1906); M. G. Lloyd, Bulletin of the Bureau of Standards, Feb. (1909).

Physical Laboratory, Teddington, Middlesex, England, has so successfully done. If in such a case great accuracy is required, the work has to be carried out with considerable care and some attention has to be paid to the fact that there is a real, if usually small, difference between the value of B corresponding to the mean H, and the mean value of B.

I have had occasion of late to determine the permeability of a small ring of extremely pure soft iron, and have found it helpful to compute by the aid of accurate HB diagrams, previously made for two or three different kinds of iron and steel in the form of long rods, what the discrepancy $(B' - B'')$ would be for these materials at different excitations, if they were made into rings of the dimensions of the one I was compelled to use. This paper gives some results which seem instructive, for a very soft kind of Norway wrought iron and for a specimen of Bessemer steel fairly typical of what one meets with in practice.

The straight rods used were magnetized and demagnetized in a uniform solenoid about five meters long, consisting of 20904 turns of well-insulated wire wound on a stout, solid-drawn brass tube through which a stream of tap water could be kept running about the rod to prevent any sensible rise of temperature. The axis of the solenoid was horizontal and perpendicular to the meridian. The flux of induction in the rods was measured by means of a test coil of fine wire wound on the rod at its centre. This coil was protected by rubber tape and its leads were insulated from the water by rubber tubes of fine bore slipped over them. The ballistic galvanometer employed had a period so long[4] that no detectable error was introduced into the readings by the fact that a measurable time was needed to make the magnetic changes incident to a reversal of current in the solenoid. The rods were demagnetized by means of a long series of currents in the solenoid, alternating in direction and gradually decreasing in intensity; and the fact that this process was successful showed that the rods were practically homogeneous throughout. The rods were so long that the corrections for the ends, as given by du Bois or by Shuddemagen,[5] were very small.

Tables I and II give the results of determinations[6] of corresponding values of H and B made by the method of ascending reversals by Mr. John Coulson and myself. A number of diagrams were obtained for each rod to make sure that the rather elaborate apparatus for demagnetizing the specimens was effective and that the metal was practically homogeneous throughout, and although the larger values of B are given in the tables rather more exactly than the observations warrant, the slow-moving ballistic galvanometers employed permitted of very accurate measurements of the flux changes in the testing coil.

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[4] Peirce, These Proceedings, **44**, 283 (1909).

[5] C. L. B. Shuddemagen, These Proceedings, **43**, 183 (1907).

[6] Peirce, American Journal of Science, **27**, 273 (1909).

After a good HB diagram, accurately drawn on a large scale, has been obtained for a given kind of iron or steel, it is possible to find out how nearly the mean value of the magnetic induction in a given ring made of this material would differ from the real induction corresponding to the mean value of the field in the metal, for any given excitation. Suppose, for example, that the ring is to be a toroid and that the radius of the circular cross section is to be a, while the centre of the section is distant c cms. from the axis, OY, of revolution of the ring. Suppose that the excitation is to be such as to make the value of H, at points distant c from OY, H_c, then the value of H at a point P (Figure 1) is $H_c \cdot c/(\mathrm{OP})$. Let the numerical value of this quantity be computed for say $n + 1$ points evenly dividing the space WV, and let the numerical values of B corresponding to these values of H be read with the help of a lens from the HB diagram. Let P represent one of the points of

TABLE I.

Annealed Norway Iron Rod 1.25 *cms. in Diameter, magnetized in a Uniform Solenoid about five Meters long. Results obtained by the Method of Reversals.*

H.	*B.*	*H.*	*B.*	*H.*	*B.*
0	0	4.0	11280	16.	15700
0.2	90	4.5	11980	18.	15900
0.4	260	5.0	12560	20.	16040
0.5	395	5.5	13000	25.	16320
0.6	570	6.0	13400	30.	16520
0.8	1125	6.5	13700	35.	16740
1.0	2150	7.0	13900	40.	16920
1.2	3160	7.5	14100	45.	17100
1.4	4140	8.0	14300	50.	17220
1.5	4600	8.5	14490	60.	17450
1.6	5200	9.0	14660	70.	17630
1.8	5825	9.5	14800	80.	17820
2.0	6600	10.	14940	90.	18020
2.5	8240	11.	15100	100.	18210
3.0	9480	12.	15360	105.	18300
3.5	10460	14.	15540		

division, let y represent the product of the value of B corresponding to the value of H at P, and the width, ST, of the ring at P, and let a curve be drawn with the y's as ordinates and the OP's as abscissas. The ratio of the area under this curve, — obtained by the help of a good Amsler's planimeter, — to πa^2, gives the mean value (B') of the magnetic induction in the ring. The average value of the field (H) is $\frac{2cH_c}{a^2}(c - \sqrt{c^2 - a^2})$ and the value (B'') of the induction corresponding to this value of H can be found from the HB diagram.

TABLE II.

Rod of Bessemer Steel 1.25 *cm. in Diameter, magnetized in a Uniform Solenoid about five Meters long. Results obtained by the Method of Reversals.*

H.	*B.*	*H.*	*B.*	*H.*	*B.*
0	0	4.5	4830	40.	15200
0.2	40	5.0	5700	50.	15720
0.4	90	5.5	6410	60.	16120
0.5	115	6.0	7060	70.	16460
0.6	145	6.5	7650	80.	16750
0.8	210	7.0	8130	90.	17000
1.0	280	7.5	8600	100.	17220
1.2	354	8.0	9040	120.	17640
1.4	426	8.5	9420	160.	18240
1.5	465	9.0	9760	200.	18800
1.6	505	9.5	10070	240.	19200
1.8	610	10.0	10390	270.	19620
2.0	730	10.5	10650	300.	19800
2.5	1180	11.	10890	350.	20240
3.0	1935	15.	12400	400.	20660
3.5	2800	20.	13360		
4.0	3760	30.	14480		

An illustration may help to make the details of the process more intelligible. Consider a toroid of the Norway iron, the circular cross section of which has a radius of one centimeter and the mean radius (c) of which is 7 centimeters. If the excitation is to be such that the value of the magnetic field at C is unity, the values of H at points distant 6, 6.1, 6.2, 6.4, 6.6, 6.8, 7.0, 7.2, 7.4, 7.6, 7.8, 8 cms. respectively from the axis of revolution of the ring, are 1.167, 1.147, 1.129, 1.094, 1.061, 1.030, 1.000, 0.972, 0.946, 0.921, 0.897, 0.875, and the values of B which correspond to these as determined from the HB diagram for the iron, are 2995, 2895, 2810, 2635, 2460, 2310, 2150, 2005, 1870, 1745, 1620, 1505. If 2895 be multiplied by the thickness of the ring at a distance of 6.1 cms. from the axis (OY), 2810 by the thickness at a distance of 6.2 cms. from OY, etc., a curve of the form KNQ shown in Figure 1 will be obtained. This curve was actually laid down on a large scale by the help of a needle point on a sheet of good coördinate paper, and the area under it was determined to be 6816, though the last significant figure is not determined. This divided by πa^2 gives 2170 as the mean value (B') of B in the ring. The mean value of H in the ring is 1.0052 and the value (B'') of B which corresponds to this is 2176. Although these results have been obtained with great care, they cannot of course be assumed to be quite correct; but it appears to be true that the error in this case is not very large.

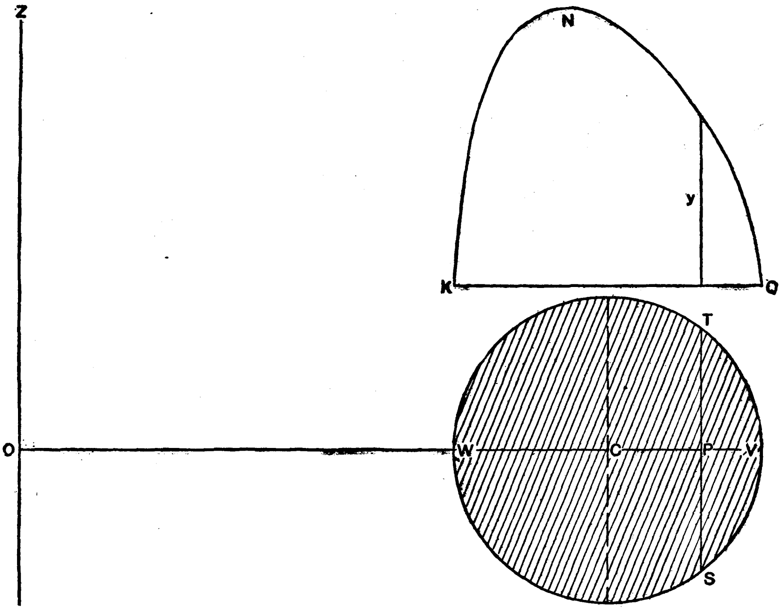

FIGURE 1.

The corresponding process in the case of a ring with rectangular cross section is much simpler and the results are more trustworthy, for the ring has a uniform thickness and the curve which bounds the nearly trapezoidal area to be measured often has so slight a curvature that the application of some form of Simpson's Rule may be made to yield a result much more accurate than a planimeter can be expected to furnish.

In the case of such a ring, as appears from the last two columns of Tables III, IV, and V, B' is usually a trifle larger than B'', for very small values of the mean H in the iron, but is equal to it for a single somewhat larger value. Then, with increasing values of H, B' is a trifle smaller than B''; but the ratio B'/B'' soon approaches unity from the under side, and, for high excitations, is sensibly equal to one. It is evident, however, that the form of B'/B'' as a function of the average value of H in the ring must depend upon the dimensions of the latter as well as upon the magnetic properties of the material of which the ring is made.

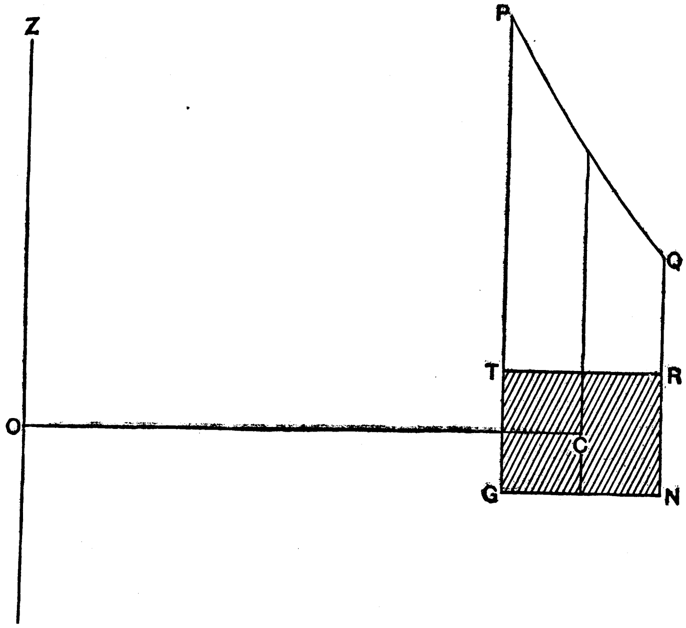

FIGURE 2.

If a ring of rectangular cross section, of the same inner and outer diameters as the toroid just described, be made of the Norway iron, and if the excitation be made such that the average value of the magnetic field in the metal at the centre (C) of the cross section is unity, the values of H and B already found may be used to draw the curve PQ, Figure 2. The area under this curve as computed by Simpson's

TABLE III.

Ring of the Pure Annealed Norway Iron. (Rectangular Cross Section. Inner Radius, 2 *a ; Outer Radius,* 3*a.)*

H_0.	H_m.	H_1.	B_0.	B_m.	B_1.	H'.	B'.	B''.
0.5	0.4	0.33	400	280	190	0.405	267	255
1.0	0.8	0.67	2150	1140	710	0.811	1235	1195
1.5	1.2	1.00	4600	3155	2150	1.216	3435	3230
2.0	1.6	1.33	6600	5020	3835	1.622	5090	5130
3.0	2.4	2.00	9480	7950	6600	2.433	7970	8040
4.0	3.2	2.67	11280	9870	8660	3.244	9910	9950
6.0	4.8	4.00	13400	12350	11280	4.866	12340	12410
8.0	6.4	5.33	14300	13630	12880	6.488	13610	13680
10.0	8.0	6.67	14940	14300	13760	8.110	14310	14330

H_0 and B_0 are the values of the magnetic force and of the induction at the inner surface of the ring; H_1, B_1 and H_m, B_m the values of the same quantities at the outer surface of the ring and at the mean radius, respectively. H' is the mean value of the magnetic field in the steel. B' is the mean value of the induction in the ring as obtained by mechanical integration from a diagram of ascending reversals for the steel, and B'' is the value of the induction corresponding to H' as shown by the same diagram. The table shows the error made by using, B'/H' instead of the exact value B''/H' for the permeability corresponding to H'.

TABLE IV.

Thinner Ring of the Annealed Norway Iron. (Rectangular Cross Section. Inner Radius, 4 *a ; Outer Radius,* 5 *a.)*

H_0.	H_m.	H_1.	B_0/B_2.	B.	H'.	B'.	B''.
0.62	0.555	0.50	1.59	405	0.558	502	490
0.94	0.833	0.75	1.90	1275	0.837	1325	1310
1.25	1.111	1.00	1.58	2702	1.116	2728	2740
2.50	2.222	2.00	1.25	7365	2.231	7386	7404
3.75	3.333	3.00	1.15	10130	3.347	10140	10150
6.25	5.555	5.00	1.08	13060	5.578	13070	13080
10.00	8.888	8.00	1.04	14600	8.924	14610	14620

TABLE V.

Ring of the Bessemer Steel. (Rectangular Cross Section. Inner Radius, 2 *a; Outer Radius,* 3 *a.)*

H_0.	H_m.	H_1.	B_0.	B_m.	B_1.	B_0/B_1.	H'.	B'.	B''.
1.0	0.8	0.67	280	201	170	1.65	0.811	209	205
2.0	1.6	1.33	725	502	385	1.88	1.622	521	510
3.0	2.4	2.00	1920	1060	725	2.65	2.433	1150	1100
4.0	3.2	2.67	3760	2260	1360	2.77	3.244	2365	2330
6.0	4.8	4.00	7060	5350	3760	1.88	4.866	5370	5470
8.0	6.4	5.33	9030	7530	6190	1.46	6.488	7550	7630
10.	8.0	6.67	10490	9030	7800	1.35	8.110	9060	9130
20.	16.	13.33	13360	12660	11830	1.06	16.219	12630	12710

Rule appears to be 2185, and the value of B corresponding to the average value 1.0069 of H is also 2185, so for these dimensions and for this particular excitation, the error represented by $B' - B''$ seems non existent.

For an excitation great enough to make the value of H at the mean radius 2, a process similar to that just described shows that B' would be 6630 and B'', 6650; but if H_c were made 5, the value of B' would be 12560 and B'', 12590. The difference in this instance is less than one quarter of one per cent of either quantity and lies within the limits of error of most magnetic measurements made upon ring specimens. For work that must be very accurate, rings much thinner than this one — in which the ratio of the outer radius to the inner radius is 4/3 — are usually employed, and the error is then practically negligible for almost all excitations.

If magnetic measurements are to be made upon rings of the dimensions sometimes used in practical permeameters, the errors arising from the difference between B' and B'' become relatively important as appears from Tables III and V, where the results for two practical cases are given. A comparison of Tables III and IV will show how fast the error decreases when the ring is made thinner.

My thanks are due to the Trustees of the Bache Fund of the National Academy of Sciences for the loan of apparatus.

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